**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.**2676**
4. 0.5
5. 0.6987

Given,

*μ* = 45 minutes,

*σ* = 8 minutes

*X = 60 minutes – 10 minutes = 50 minutes*

*We have z = (X-μ) /σ = (50 – 45)/8 = 0.625*

*P(x>50) = 1- 0*.73237 = 0.26763 = 26.76%

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44

Ans:- False

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans:- True

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans:-

Here, 2X1 –(X1 +X2)= X1+ X1 –X1 –X2= X1-X2 ~ *N*(μ1- μ2, σ12-σ22)

The distribution is normal with Mean = μ1- μ2 and Variance = σ12-σ22

1. Let X ~ N (100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. **48.5, 151.5**
6. 90.1, 109.9

Ans: - Given,

µ = 100

σ = 20

*Here,* 1-α /2= (1-0.99)/2 = 0.005

Z1 *= -2.58 (from Z-table)*

Z2= 2.58

Now,

*Z = (x-μ) /*σ

*a = μ +* Z1σ = 100 -2.58 \*20 = 48.4

*b = μ +* Z2 σ = 100+ 2.58\*20 = 151.6

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N (5, 32) and Profit2 ~ N (7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: - Total Profit of the company = Profit1 +Profit2 ~ N (5+7, 32 + 42)

Profit ~ N (12, 52)

Here,

µ= 12 $ Million,

σ = 5 $ Million

*Here,* 1-α /2= (1-0.95)/2 = 0.025

Z1 = -1.96 (for 0.025)

Z2 = 1.96 (for 0.975)

*a = μ +* Z1 σ = 12 -1.96 \*5 = 2.2 \*42 =Rs92.4 million

*b = μ +* Z2σ = 12+ 1.96\*5 = 21.8 \*42 = Rs. 915.6 million

The range is Rs. 92.4 million to Rs. 915.6 million

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans:-

Z score for 5th percentile = -1.64

*X = μ +* Zσ = 12 -1.64 \*5 = 3.8 = 3.8\* 42 = Rs.159.6 million

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans:-

a.) Case 1: - Profit1 ~ N (5, 32)

Profit Zone

*a = μ +* Z1 σ = 5 -1.96 \*3 = -0.88

*b = μ +* Z2σ = 5+ 1.96\*3 = 10.79

b.) Case 2: - Profit2 ~ N (7, 42)

*a = μ +* Z1 σ = 7 -1.96 \*4 = -0.84

*b = μ +* Z2σ = 7+ 1.96\*4 = 14.84

The profit zone is larger in case2 than in case 1, So loss is more in case1